USN | | | | | | | | | | **10MAT21**

Second Semester B.E. Degree Examination, January 2013

Engineering Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions only on OMR sheet page 5 of the answer booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose correct answers for the following : **(04 Marks)**

i) The general solution of the equation $p^2 - 5p + 6 = 0$ is : A) $(y - 2x - c)(y - 3x - c) = 0$
B) $(y + 2x - c)(y + 3x - c) = 0$ C) $(y - 2x - c)(y + 3x - c) = 0$ D) $(y - x - c)(y + x - c) = 0$

ii) If a differential equation is solvable for y then it is of the form
A) $x = f(y, p)$ B) $y = f(x, p)$ C) $y = f(x^2, py)$ D) $x = f(y^2, p)$

iii) The differential equation of the form $y = px + f(p)$ whose general solution is $y = cx + f(c)$ is known as
A) Glairaut's equation B) Cauchy's equation C) Lagrange's equation D) None of these

iv) The singular solution of the equation $y = px - \log p$ is
A) $y = 1 - \log x$ B) $y = 1 - \log(1/x)$ C) $y = \log x - 2x$ D) none of these

b. Solve the equation $p^2 + p(x + y) + xy = 0$. **(04 Marks)**

c. Solve the equation $xp^2 - 2yp + ax = 0$. **(06 Marks)**

d. Obtain the general solution and singular solution of the equation $\sin px \cos y = \cos px \sin y + p$. **(06 Marks)**

2 a. Choose correct answers for the following : **(04 Marks)**

i) The homogeneous linear differential equation whose auxiliary equation has roots 1, 1, -2 is
A) $D^3 + 3D^2 + D + 1 = 0$ B) $D^3 - 3D + 2 = 0$ C) $(D + 1)^2(D + 2) = 0$ D) $D^3 + 3D + 2 = 0$

ii) The complementary function for the differential equation $(D^2 + 2D + 1)y = 2x + x^2$ is
A) $c_1e^{-x} + x^2c_2e^{-x}$ B) $c_1e^x + c_2e^{-x}$ C) $(c_1 + c_2)e^x$ D) $(c_1 + c_2)e^{-x}$

iii) The particular integral of $(D^2 + a^2)y = \cos ax$ is
A) $(-x/2a)\sin ax$ B) $(x/2a)\cos ax$ C) $(-x/2a)\cos ax$ D) $(x/2a)\sin ax$

iv) The general solution of an n^{th} order linear differential equation contains : A) at most n constants, B) exactly n independent constants, C) at least n independent constants, D) more than n constants.

b. Solve: $y'' - 2y' + y = xe^x \sin x$. **(04 Marks)**

c. Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \cos x + 4$. **(06 Marks)**

d. Solve : $dx/dt = 2x - 3y$, $dy/dt - y - 2x$ given $x(0) = 8$ and $y(0) = 3$. **(06 Marks)**

3 a. Choose correct answers for the following : **(04 Marks)**

i) By the method of variation of parameters, the value of W is called
A) the Demorgan's function B) Euler's function C) Wronskian of the function D) none of these

ii) The differential equation of the form $a_0(ax + b)^2 y'' + a_1(ax + b)y' + a_2y = \phi(x)$ is called
A) Simultaneous equation B) Legendre's equation C) Cauchy's equation D) Euler's equation

iii) The equation $x^3\frac{d^3y}{dx^3} + 3x^2\frac{dy}{dx^2} + x\frac{dy}{dx} = x^3 \log x$ by putting $x = e^t$ with $D = d/dt$ reduces to
A) $(D^3 + D^2 + D)y = 0$ B) $D^3y = 0$ C) $D^3y = te^{3t}$ D) none of these

iv) To find the series solution for the equation $4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$, we assume the solution as
A) $y = \sum_{r=0}^{\infty} a_r x^{K+r}$ B) $y = \sum_{r=0}^{\infty} a_r x^r$ C) $y = \sum_{r=0}^{\infty} a_{r+1} x^{r+1}$ D) $y = \Sigma(ax + b)x^r$

b. Using the variation of parameters method, solve the equation $y'' - 2y' + y = e^x/x$. **(04 Marks)**

c. Solve the equation $x^2y'' - xy' + 2y = x \sin(\log x)$. **(06 Marks)**

d. Obtain the Frobenius type series solution of the equation $x\frac{d^2y}{dx^2} + y = 0$. **(06 Marks)**

4 a. Choose correct answers for the following : **(04 Marks)**

i) The partial differential equation obtained by eliminating arbitrary constants from the relation $Z = (x - a^2) + (y - b)^2$ is
A) $p^2 + q^2 = 4z$ B) $p^2 - q^2 = 4z$ C) $p + q = z$ D) $p - q = 2z$

ii) The auxiliary equations of Lagrange's linear equation $Pp + Qq = R$ are
A) $dx/p = dy/q = dz/R$ B) $dx/P = dy/Q = dz/R$ C) $dx/x = dy/y = dz/z$ D) $dx/x + dy/y + dz/z = 0$

iii) General solution of the equation $\frac{\partial^2 z}{\partial x \partial y} = x^2y$ is
A) $(1/6)x^3y^2 + f(y) + g(x)$ B) $(1/6)x^3y^2 + f(y)$ C) $(1/6)x^3y^3$ D) none of these

iv) By the method of separation of variables, we seek a solution in the form
A) $X = X(x)Y(y)$ B) $Z = X + Y$ C) $Z = X^2Y^2$ D) $Z = X/Y$

b. Form a partial differential equation from the relation $Z = f(y) + \phi(x + y)$.

c. Solve the equation $(x^2 - y^2 - z^2)p + 2xy\,q = 2xz$.

d. Use the method of separation of variables to solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ given that $u(x, 0) = 6e^{-3x}$.

PART – B

5 a. Choose correct answers for the following : **(04 Marks)**

i) $\int_0^1\int_0^{x^2} e^{y/x}\,dydx$ is equal to: A) 1/2 B) -1/2 C) 1/4 D) 2/5

ii) The integral $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$ by changing to polar form becomes

A) $\int_{\theta=0}^{\pi/2}\int_{r=0}^{\infty} e^{r^2}\, r\,drd\theta$ B) $\int_{\theta=0}^{\pi/2}\int_{r=0}^{\infty} e^{-r^2}\, r\,drd\theta$ C) $\int_{\theta=0}^{\pi/2}\int_{r=0}^{a} e^{2r}\,drd\theta$ D) none of these

iii) $\beta(3, \frac{1}{2})$ is equal to: A) 16/11 B) 16/15 C) 15/16 D) $2\pi/3$

iv) The integral $2\int_0^\infty e^{-x^2}dx$ is : A) $\Gamma(3/2)$ B) $\Gamma(n+1)$ C) $\Gamma(-1/2)$ D) $\Gamma(1/2)$

b. Evaluate by changing the order of integration $\int_0^a\int_0^{2\sqrt{xa}} x^2dydx$, $a > 0$. **(04 Marks)**

c. Evaluate the integral $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\,dzdydx$. **(06 Marks)**

d. Prove that $\int_0^\infty xe^{-x^8}dx \times \int_0^\infty x^2e^{-x^4}dx = \frac{\pi}{16\sqrt{2}}$. **(06 Marks)**

6 a. Choose correct answers for the following : **(04 Marks)**

i) If $f = (5xy - 6x^2)i + (2y - 4x)j$ then $\int_C f.dr$ where c is the curve $y = x^3$ from the points (1, 1) to (2, 8) is

A) 35 B) -35 C) 3x + 4y D) none of these

ii) In Green's theorem in the plane $\int_C (Mdx + Ndy) =$ ______.

A) $\iint_A\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dxdy$ B) $\iint_A\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dx$ C) $\iint_A\left(\frac{\partial M}{\partial x}-\frac{\partial N}{\partial y}\right)dydx$ D) $\iint_A\left(\frac{\partial M}{\partial x}-\frac{\partial N}{\partial y}\right)dxdy$

iii) If $\int_C f\cdot d\vec{r} = 0$ then f is called: A) rational B) irrotational C) solenoidal D) rotational

iv) If all the surfaces are closed in a region containing volume V then the following theorem is applicable
A) Stoke's theorem B) Green's theorem C) Gauss divergence theorem D) none of these

b. If $f = (2x^2 - 3z)i - 2xy\hat{j} - 4x\hat{k}$, evaluate $\int_v \text{curl } f\,dv$ where v is the volume of the region bounded by the planes x = 0, y = 0, z = 0 and 2x + 2y + z = 4. **(04 Marks)**

c. Verify Green's theorem for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the triangle formed by x = 0, y = 0 and x + y = 1. **(06 Marks)**

d. Verify the Stokes's theorem for $f = -y^3\hat{i} + x^3\hat{j}$ where s is the circular disc $x^2 + y^2 \le 1$, z = 0. **(06 Marks)**

7 a. Choose correct answers for the following : **(04 Marks)**

i) The Laplace transform of f(t)/t when L[f(t)] = F(s) is: A) $\int_0^\infty F(s)ds$, B) $\int_s^\infty F(s)ds$, C) $\int_0^\infty F(s-a)ds$, D) $\int_0^\infty F(s+a)ds$

ii) $L[t^3e^{2t}] =$ ______. A) $(3!)/(s-2)^4$ B) $(3!)/(s+2)^4$ C) $3/(s-2)^4$ D) $3/(s-2)$

iii) $L\{f(t - a)H(t - a)\}$ is equal to : A) $e^{-as}L\{f(t)\}$ B) $e^{as}L\{f(t)\}$ C) $(e^{-as})/s$ D) $[L\{f(t)\}]/se^{-as}$

iv) $L\{\delta(t)\}$ is equal to : A) 0 B) -1 C) e^{-as} D) L

b. Evaluate $L\{\sin t \sin 2t \sin 3t\}$. **(04 Marks)**

c. A periodic function of period $2\pi/\omega$ is defined by $f(t) = \begin{cases} E\sin\omega t & \text{for } 0 \le t \le \pi/\omega \\ 0 & \text{for } \pi/\omega \le t \le 2\pi/\omega \end{cases}$. Find $L\{f(t)\}$. **(06 Marks)**

d. Express $f(t) = \begin{cases} 2t & 0 < t \le \pi \\ 1 & t > \pi \end{cases}$ in terms of unit step function and hence find $L\{f(t)\}$. **(06 Marks)**

8 a. Choose correct answers for the following : **(04 Marks)**

i) $L^{-1}\{F(s)/s\}$ is equal to : A) $\int_0^t f(t)dt$ B) $\int_0^\infty f(t)dt$ C) $\int_0^\infty f(t-a)dt$ D) $\int_0^t f(t-a)dt$

ii) $L^{-1}\{1/(s^2 + 2s + 5)\}$ is equal to : A) $e^t \sin 2t$ B) $1/2\, e^{-t}\sin 2t$ C) $1/2\, e^t \cos 2t$ D) $e^2t\cos 2t$

iii) $f(t) * g(t)$ is defined by: A) $\int_0^t f(t-u)g(u)du$ B) $\int_0^\infty f(t)g(t)dt$ C) $\int_0^t f(t)g(t)du$ D) $\int_0^t f(u)g(u)du$

iv) $L^{-1}\{1/(s^2 + a^2)\}$ is : A) cos at B) sec at C) sin at D) (1/a) sin at

b. Find $L^{-1}\{(2s - 1)/(s^2 + 2s + 17)\}$. **(04 Marks)**

c. By employing the convolution theorem evaluation $L^{-1}\{s/(s^2 + a^2)^2\}$. **(06 Marks)**

d. Solve the initial value problem $y'' - 3y' + 2y = 4t + e^{3t}$, $y(0) = 1$, $y'(0) = -1$ using Laplace transforms. **(06 Marks)**

* * * * *